

Analytical approach for non-linear Zakharov-Kuznetsov differential equation by fractional reduced transformation method

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Abstract

This research study aims to determine a novel and modest algorithm for weak, nonlinear ion-acoustic waves in a highly magnetized, lossless plasma in 2 dimensions, namely the Zakharov-Kuznetsov equation, using the fractional reduced-differential transformation method (FRDTM). This approach is an advanced modification of the Laplace-transform algorithm to simplify the calculation. FRDTM has to establish the analytical solution or closed numerical solution for any kind of ordinary, partial and fractional differential equations. Numerical results, accompanied by graphical demonstrations, show that the proposed approach provides reliable, accurate results and offers a new explanation for a different type of dynamical system.

Keywords: Fractional calculus, Approximate solution, Zakharov-Kuznetsov equation, Non-linear fractional, fractional differential equations, reduced differential transformation method (RDTM).

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1. Introduction

After the development of the past decade, greater attention from researchers towards the fractional differential equation, due to their capability to improve real-life problems, applies to many fields of engineering and sciences. Fractional calculus plays an important role in various physical phenomena in engineering science by successfully illustrating how to develop the models. The fractional differential calculus applied in various fields of applied mathematics and physics like viscoelasticity, electrochemistry, acoustics, chemical, and signal processing. Baleanu, et al., (2009); Debnath, (2003); Herzallah, et al., (2010); Hilfer, (2000); Mainardi et al., (2001); Miller and Ross, (1993); Podlubny, (1999); Saker, et al., (2012); Srivastava, et al., (2014); Young, (1995); He, (1999, 1998), Numerous researchers have paid much attention in the recent year to the formulation of linear and non-linear partial differential calculation of using several approaches to combine with Laplace-transform. For the fractional calculus, Srivastava et al., (2013, 2014) discussed the two or three dimensions solution of telegraphic equations. Srivastava et al., (2014) analysed the biological population model of 2D fractional-order time. Shukla et al. (2014) discuss the result of the time-fractional order Cauchy-diffusion equation and show that FRDTM is a semi-analytical method that is easy to handle and provides the analytical solution for the linear and dynamical equations.

Here, the Fractional Zakharov-Kuznetsov equation (FZK) is studied in the following form.

$$\frac{\partial^\beta u}{\partial t^\beta} + a_1 \frac{\partial u^p}{\partial x} + a_2 \frac{\partial^3 u^q}{\partial x^3} + a_3 \frac{\partial^3 u^r}{\partial x \partial y^2} = 0. \quad (1)$$

Here, u is the function of independent variables t , x , and y .

β = denote the parameter which defines the fractional order derivative, ($0 < \beta \leq 1$).

$p, q, r \neq 0 \in I(\text{Integers})$, and a_1, a_2, a_3 are arbitrary constants.

The Korteweg-de Vries equations have an essential role in the application of engineering and sciences, and it one of disreputable variation is found by the comparison of Zakharov-Kuznetsov. Korteweg and Vries (1985) explore Fractional Zakharov-Kuznetsov (FZK) equation is two-dimensional study of the Korteweg-de Vries equation, obtained in an anisotropic situation which is a perfect example of rotational liquid, where the latitudinal variation, necessity of the rotation degree induces anisotropy among the southern and the longitudinal directions. Furthermore, inconsistency with the Kadomtsev-Petviashvili equation, the Zakharov-Kuznetsov equations (FZK) supports steady lump solitary waves.

Most researchers in the current days are wildly using the theory and the concept of fractional calculus and the Zakharov-Kuznetsov equation. Kumar et al., (2014) study of numerically compute of non-linear fractional Zakharov-Kuznetsov equation present in ion-acoustic waves by Homotopy Perturbation Transform Method (HPTM). Senol et al., (2018) discussed an approach for ZK Equations are continuous power-series procedure and iteration of perturbation algorithm. Prakash et al., (2018) using a new Sumudu-transform method (NISTM) for Non-linear fractional ZK equation in 2-Dimension.

However, This Research paper is ordered as follows: Section 1 is the Introduction, while the necessary preliminaries and definition of fractional-calculus theory considered and used for further study is in Section 2 and in Section 3, the basic idea and objective of the fractional reduced-differential transformation method (FRDTM) is reviewed and explain to resolve the non-linear

fractional Zakharov-Kuznetsov. Section 4 presents solutions to two test problems for the fractional Zakharov-Kuznetsov equation. Section 5 presents the results and discussion, including a comparison of available solutions, and Section 6 is the concluding part.

2. Theory of Fractional Calculus

This section covers some preliminary and notations which were considered for exploring this study. The great mathematicians contributed to its development of fractional calculus directly or indirectly, included Lacroix, Laplace, Euler, Fourier, Riemann, Liouville, and Abel. Lacroix (1819), the first-mathematician to explore the study of fractional derivative, but Abel (1965) in 1802-1829, solve of an integral equation by using fractional calculus, and Explores the formulation of the problem to describe the shape of a curve in such a way that the decline time an object sliding down the curve at uniform gravity is independent of the starting point of the object, this problem is called tautochrone or isochrone curve problem. In the existing literature, fractional-calculus is about 20 decades old concept and numerous descriptions and facts about fractional calculus, fractional derivatives, and integrals have proposed. However, the consistency of the exact definition is because of Liouville's contribution as shown below.

Definition 2(a): "A real function $f(x)$, $x > 0$ is said to be in the space C_μ , $\mu \in \mathbb{R}$ if there exists a real number $q (> \mu)$, such that $f(x) = x^q g(x)$, where $g(x) \in C[0, \infty)$, and it is said to be in the space C_μ^m if $f(x)^m \in C_\mu$, $m \in \mathbb{N}$ ".

In an existing literature of fractional calculus Riemann-Liouville description has most popular concepts. The Riemann-Liouville interpretation of a fractional differential, however, allows for the same results as Lacroix achieved in the equation

Definition 2 (b): "For a function f , the Riemann-Liouville fractional integral operator of order α

≥ 0 , is defined as $J^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) dt$, $\alpha > 0, x > 0$, $J^0 f(x) = f(x)$.,

With a fractional-differential equation, when trying to form a real-world problem, Millar and Ross (1993) state that the Riemann-Liouville derivative has many difficulties. Caputo and Mainardi (1971) describe modified-fractional derivative operator D^α , The caputo-fractional derivative extend the application for initial boundary condition in his work on the viscous-elasticity principle to eliminate the incoherence.

3. Fractional Reduced Differential Transform Method (FRDTM)

This part examined the fundamental concept of the "reduced-differential transformation method" (RDTM), a study conducted $h(\xi, t)$ for two variables and proposes that this can be interpreted as a combination of two function of one-variable, i.e. the function can be described as $h(\xi, t) = F(\xi).G(\xi)$, based on the characteristics of the one-dimensional differential transformation, the function can be describe as

$$h(\xi, t) = \sum_{i=0}^{\infty} \xi^i F(i) \sum_{j=0}^{\infty} G(j) t^j = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} H(i, j) \xi^i t^j \quad (2)$$

Where, $H(i, j)$ is the product of $F(i)$ and $G(i)$, called the spectrum of $h(\xi, t)$.

Definition 3 (a): If $H_k(\xi)$ is analytic with respect to ξ (space variable) and t (time variable) in the domain, the t -dimensional spectrum function as

$$H_k(\xi) = \frac{1}{\Gamma(\beta k + 1)} \left(\frac{\partial^k}{\partial t^k} h(\xi, t) \right)_{t=t_0} \quad (3)$$

Shows the reduced transformed function of $h(\xi, t)$, here k consider as the order of the time fractional-derivative. During the study, $H_k(\xi)$ stands for the reduced transformed function while $h(\xi, t)$ represents the original function, the inverse-differential transformation of $H_k(\xi)$ define as

$$h(\xi, t) = \sum_{k=0}^{\infty} H_k(\xi) \cdot (t - t_0)^{\beta k} \quad (4)$$

From equation (2) and (3) we get , $h(\xi, t) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k\beta + 1)} \left(\frac{\partial^k}{\partial t^k} h(\xi, t) \right)_{t=t_0} (t - t_0)^{k\beta}$ (5)

at $t = 0$ we get, $h(\xi, t) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k\beta + 1)} \left[\frac{\partial^k}{\partial t^k} h(\xi, t) \right]_{t=t_0} t^{k\beta}$ (6)

The reduce differential transform theory is the outcome of the expansion of a function in the power-series.

Definition 3(b): “The $v(\xi, t) = R^{-1}[V_k(\xi)]$ & $u(\xi, t) = R^{-1}[U_k(\xi)]$, the multiplication \otimes represent the reduced-differential transform type of the convolution, then the basic algebraic operations for the fractional reduced differential transform method are shown in Table.1, the gamma function is the continuous expansion of factorial function and is define as

$$\overline{(m)} = \int_0^{\infty} (t^{m-1} e^{-z}) dz ; m \in \mathbb{C} \text{ and } \overline{(m+1)} = m \overline{(m)} = m!$$

Table.1 Basic Operations of Fractional Reduced Differential Transform Method (FRDTM)

Original Function	Function after FRDTM
$u(\xi, t) \cdot v(\xi, t)$	$U_k(\xi) \otimes V_k(\xi) = \sum_{r=0}^k U_r(\xi) V_{k-r}(\xi)$
$\alpha \cdot u(\xi, t) \pm \beta \cdot v(\xi, t)$	$\alpha \cdot U_k(\xi) \pm \beta \cdot V_k(\xi)$
$\frac{\partial^h}{\partial x^h} u(\xi, t)$	$\frac{\partial^h}{\partial \xi^h} U_k(\xi)$

$\frac{\partial^{N\beta}}{\partial t^{N\beta}} u(\xi, t)$	$\frac{\Gamma(k\beta + N\beta + 1)}{\Gamma(k\beta + 1)} U_{k+N}(\xi)$
$\xi^m t^n u(\xi, t)$	$x^m U_{k-n}(\xi)$
$e^{\lambda t}$	$\frac{\lambda^k}{k!}$
$Sin(wt + \alpha)$	$\frac{w^k}{k!} Sin\left(\frac{\pi k}{2!} + \alpha\right)$
$Cos(wt + \alpha)$	$\frac{w^k}{k!} Cos\left(\frac{\pi k}{2!} + \alpha\right)$

4. Alternative approach of Fractional Zakharov-Kuznetsov equation

In order to determine the accuracy and applicability of the FRDTM, apply the suggested alternative method of Fractional Zakharov-Kuznetsov equation by presenting a numerical example for different values of p, q, r in Fractional Zakharov-Kuznetsov equation, providing verification of RDTM's efficiency and consistency.

Illustration.1 We consider an illustration, which follows the Fractional Zakharov-Kuznetsov equation

$$\frac{\partial^\beta u}{\partial t^\beta} + \frac{\partial u^2}{\partial \xi} + \frac{\partial^3 u^2}{8 \partial \xi^3} + \frac{\partial^3 u}{8 \partial \xi \partial \psi^2} = 0 \tag{7}$$

With $(p, q, r) \in I$, which takes the value (2, 2, 2) respectively and $0 < \beta \leq 1$

The analytical solution of such existing problem when $\beta = 1$ and with the initial condition

$$u(\xi, \psi, 0) = \left(\frac{4\rho \sinh^2(\xi + \psi)}{3} \right) \text{ and } u(\xi, \psi, t) = \left(\frac{4\rho \sinh^2(\xi + \psi - \rho t)}{3} \right), \text{ here } \rho \text{ is an}$$

arbitrary constant.

By applying the Reduced Differential Transform Method (RDTM), we get

$$U_0 = \frac{\rho(e^{(\xi+\psi)} - e^{-(\xi+\psi)})^2}{3}, \text{ for the value of } k = 0, \tag{8}$$

$$U_1 = \left(-\frac{2^2 \rho}{3 \beta!} \right) (e^{2(\xi+\psi)} - e^{-2(\xi+\psi)}), \text{ for the value of } k = 1, \tag{9}$$

$$U_2 = \left(-\frac{2^5 \rho^2 \beta!}{3^2 2 \beta!} \right) (-2e^{2(\xi+\psi)} - 2e^{6(x+\psi)} + 5e^{8(\xi+\psi)} - 1), \text{ for the value of } k = 2, \tag{10}$$

$$U_3 = -\frac{2^6 \rho^2 e^{-6(\xi+\psi)}}{243(\beta!)^2 (2\beta)!(3\beta)!} (15360\rho^4 e^{12(\xi+\psi)} (\beta!)^4 - 512\rho^3 e^{2(\xi+\psi)} (e^{8(\xi+\psi)} - 1)(\beta)!(2\beta)! - 18\rho(148e^{2(\xi+\psi)} + 33e^{4(\xi+\psi)} - 8e^{6(\xi+\psi)} - 17e^{8(\xi+\psi)} - 20e^{10(\xi+\psi)} + 5e^{12(\xi+\psi)} - 5)(\beta!)^3 (2\beta)! + 27e^{2(\xi+\psi)} (1 - e^{8(\xi+\psi)}))(2\beta!)^2 \quad (11)$$

Proceed in the same procedure the rest of components $U_k(x)$ by FRDTM can be found.

$$u(\xi, t) = \sum_{k=0}^{\infty} U_k(\xi) t^{\beta k} = \sum_{k=0}^{\infty} U_k(\xi) t^{k/\lambda} \quad (12)$$

Where $\beta = \frac{1}{\lambda}$, $\lambda > 0$ applying the inverse-differential reduced transform of $U_k(\xi)$, Thus we obtained $u(\xi, t)$ which is the solution of the Equation (7) given as:

$$u(\xi, t) = U_0(\xi) + U_1(\xi) t^{1/\lambda} + U_2(\xi) t^{2/\lambda} + U_3(\xi) t^{3/\lambda} \dots \quad (13)$$

Illustration 2. We consider an illustration, which follow Fractional Zakharov-Kuznetsov equation

$$\frac{\partial^\beta \mathbf{u}}{\partial t^\beta} + \frac{\partial}{\partial \xi}(\mathbf{u}^3) + \frac{2\partial^3}{\partial \xi^3}(\mathbf{u}^3) + \frac{2\partial^3}{\partial \xi \partial \psi^2}(\mathbf{u}^3) = 0 \quad (14)$$

With $(p, q, r) \in I$, which takes the value (3, 3, 3) respectively and $0 < \beta \leq 1$

The analytical solution of problem when $\beta = 1$ and under the initial condition

$$u(\xi, \psi, 0) = \frac{3}{2} \rho \sinh\left[\frac{1}{6}(\xi + \psi)\right], \text{ Where } \rho \text{ is an arbitrary constant,}$$

$$\text{and } u(\psi, t) = \frac{3}{2} \rho \text{Sinh}\left(\frac{1}{6}(\xi + \psi - \rho t)\right)$$

On apply Reduced-Differential Transform Method, we get

$$U_0 = \frac{3\rho}{2} \left(\frac{e^{\frac{(\xi+\psi)}{6}} - e^{-\frac{(\xi+\psi)}{6}}}{2} \right), \text{ for the value of } k = 0 \quad (15)$$

$$U_1 = -\frac{3^3 \rho^3}{2^6 (\beta!)} \left[e^{\frac{(\xi+\psi)}{2}} + e^{-\frac{(\xi+\psi)}{2}} - \frac{5}{9} \left(e^{\frac{(\xi+\psi)}{6}} + e^{-\frac{(\xi+\psi)}{6}} \right) \right], \text{ for the value of } k = 1 \quad (16)$$

$$U_2 = -\frac{3^5 \rho^5}{2^{10} \times (2\beta)!} \left[\frac{5 \times 2^3}{3^3} \left(e^{\frac{(\xi+\psi)}{6}} - e^{-\frac{(\xi+\psi)}{6}} \right) - 11 \left(e^{\frac{(\xi+\psi)}{2}} - e^{-\frac{(\xi+\psi)}{2}} \right) + \frac{17 \times 5}{3^2} \left(e^{\frac{5(\xi+\psi)}{6}} - e^{-\frac{5(\xi+\psi)}{6}} \right) \right], \text{ for the value of } k = 2 \tag{17}$$

$$U_3 = -\frac{3^7 \rho^7 (2\beta)!}{2^{14} (3\beta)!} \left[\left[\frac{1}{(2\beta)!} \left\{ \frac{889}{54} \left(e^{\frac{(\xi+\psi)}{2}} + e^{-\frac{(\xi+\psi)}{2}} \right) - \frac{139}{54} \left(e^{\frac{(\xi+\psi)}{6}} + e^{-\frac{(\xi+\psi)}{6}} \right) - \frac{1345}{54} \left(e^{\frac{5(\xi+\psi)}{6}} + e^{-\frac{5(\xi+\psi)}{6}} \right) + \frac{595}{54} \left(e^{\frac{7(\xi+\psi)}{6}} + e^{-\frac{7(\xi+\psi)}{6}} \right) \right\} + \frac{1}{(\beta!)^2} \left\{ \frac{7}{6} \left(e^{\frac{7(\xi+\psi)}{6}} + e^{-\frac{7(\xi+\psi)}{6}} \right) - \frac{95}{54} \left(e^{\frac{5(\xi+\psi)}{6}} + e^{-\frac{5(\xi+\psi)}{6}} \right) + \frac{277}{486} \left(e^{\frac{(\xi+\psi)}{6}} + e^{-\frac{(\xi+\psi)}{6}} \right) + \frac{15}{162} \left(e^{\frac{(\xi+\psi)}{2}} + e^{-\frac{(\xi+\psi)}{2}} \right) \right\} \right] + 4 \left[\frac{1}{(2\beta)!} \left\{ \frac{889}{216} \left(e^{\frac{(\xi+\psi)}{2}} + e^{-\frac{(\xi+\psi)}{2}} \right) - \frac{139}{1944} \left(e^{\frac{(\xi+\psi)}{6}} + e^{-\frac{(\xi+\psi)}{6}} \right) - \frac{33625}{1944} \left(e^{\frac{5(\xi+\psi)}{6}} + e^{-\frac{5(\xi+\psi)}{6}} \right) + \frac{29155}{1944} \left(e^{\frac{7(\xi+\psi)}{6}} + e^{-\frac{7(\xi+\psi)}{6}} \right) + \frac{1}{(\beta!)^2} \left\{ \frac{343}{216} \left(e^{\frac{7(\xi+\psi)}{6}} + e^{-\frac{7(\xi+\psi)}{6}} \right) - \frac{2375}{1944} \left(e^{\frac{5(\xi+\psi)}{6}} + e^{-\frac{5(\xi+\psi)}{6}} \right) + \frac{277}{17496} \left(e^{\frac{(\xi+\psi)}{6}} + e^{-\frac{(\xi+\psi)}{6}} \right) + \frac{15}{648} \left(e^{\frac{(\xi+\psi)}{2}} + e^{-\frac{(\xi+\psi)}{2}} \right) \right\} \right\} \right] \right] \tag{18}$$

Proceeding in the similar way the remaining components $U_k (\xi)$ by FRDTM is obtained.

$$u(\xi, t) = \sum_{k=0}^{\infty} U_k (\xi) t^{\beta k} = \sum_{k=0}^{\infty} U_k (\xi) t^{k/\lambda} \tag{19}$$

Where $\beta = \frac{1}{\lambda}$, $\lambda > 0$, By the inverse differential reduced transform of $U_k (x)$, thus obtain a solution $u(x, t)$ of the Equation (14) provide as:

$$u(\xi, t) = U_0 (\xi) + U_1 (\xi) t^{1/\lambda} + U_2 (\xi) t^{2/\lambda} + U_3 (\xi) t^{3/\lambda} \dots \tag{20}$$

5. Result and Discussion

The solution of first problem in Figure. 1 and Figure. 2 for the values fractional derivative *i.e* of beta 0.5 and 0.75 is constant till 0.3 and 0.5 of *x* and *y* coordinate respectively for the function. And then after diereses the values of ξ and ψ function is convergent. In Figure 3, the order of a differential equation is one and the solution will be constant at 0.8 and above. The error has shown between the analytical solution and Numerical solution in figure 4, it validate that reduced-differential transform method (RDTM) has enhanced the homotopy-perturbation transformation method. This method provide an analytical solution, with easily computable in the form of a convergent series.

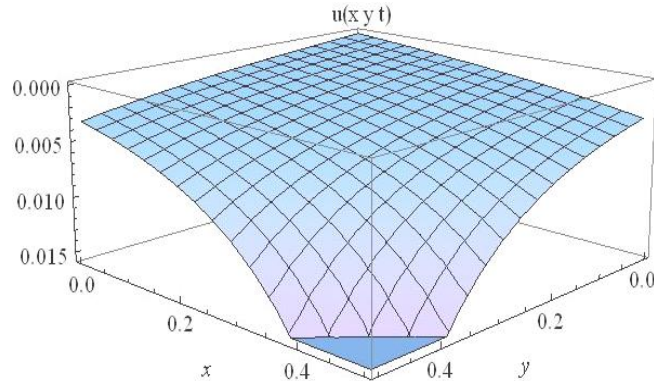


Figure 1 Series Solution of first illustration for the values of $\beta = 0.5, \rho = 0.001,$
 ξ and ψ from 0 to 1

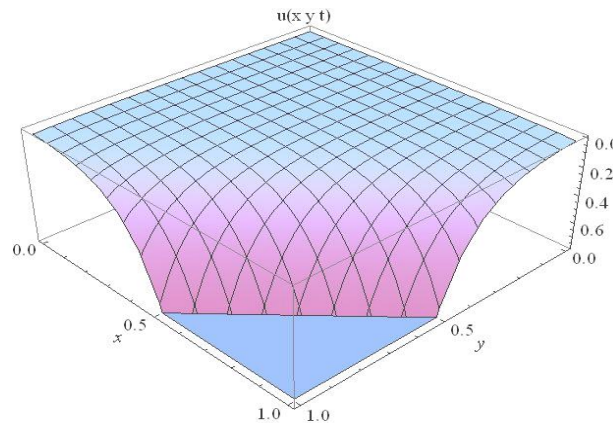


Figure 2 Series solution of first illustration for the values of $\beta = 0.75, \rho = 0.001,$
 ξ and ψ from 0 to 1

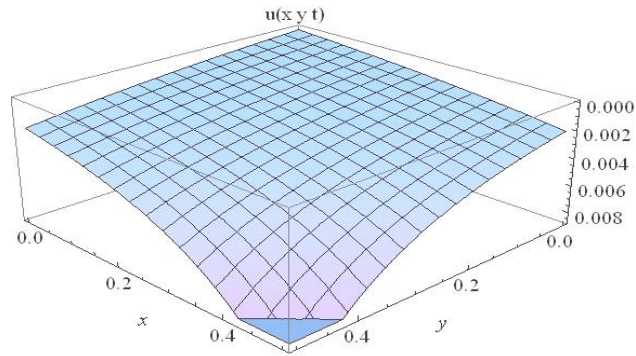


Figure 3 Series solution of the first illustration for the values of $\beta = 1, \rho = 0.001$, ξ and ψ from 0 to 1

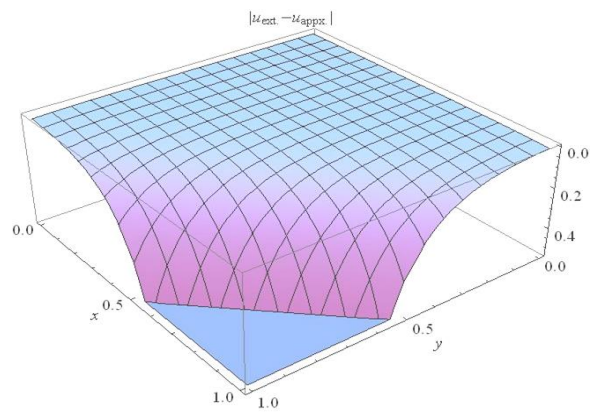


Figure 4 Error of approximate and exact solution in the first illustration

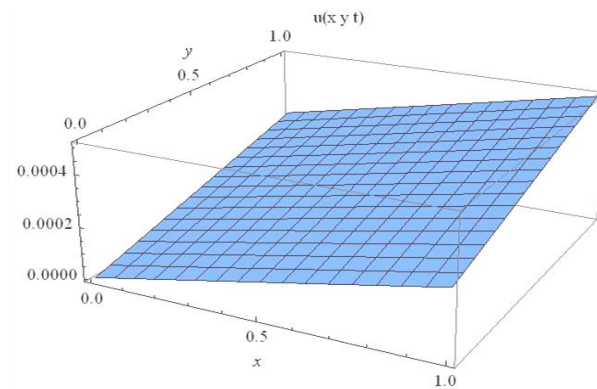


Figure 5 The series solution of the second illustration graphically for the values of $\beta = 0.5, \rho = 0.001$ and ξ and ψ from 0 to 1

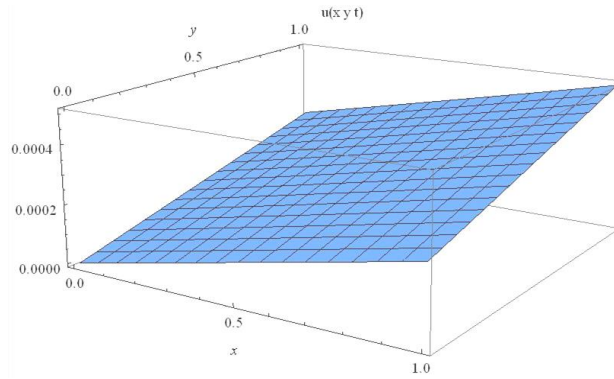


Figure 6 The Series Solution of the second illustration graphically for the values of $\beta = 1, \rho = 0.001, \xi$ and ψ from 0 to 1

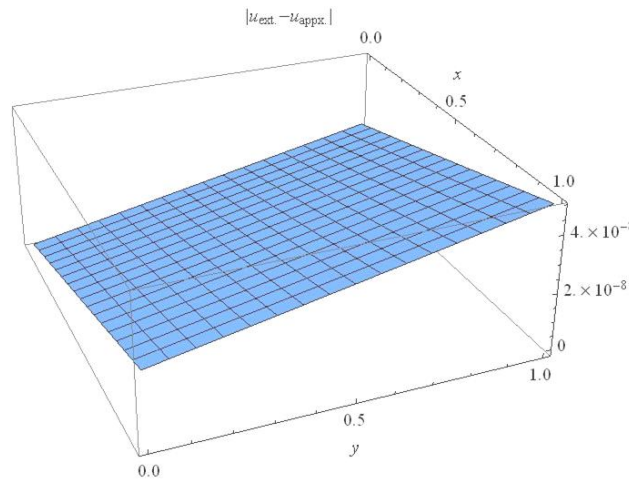


Figure 7 Error of approximate and exact solution of the second illustration

The solution for the second illustration is a plot form Figure 5 to Figure 7, where the first two graph shows the result for the fractional and non-fractional derivative. In Figure 7, plot the graph for the error for the exact and approximate solution, and we found that the error is decreasing as the values of the variables increase. Thus, it shows that the Reduced Differential Transform Method gives an improved result, instead of the homotopy perturbation transformation method.

6. Conclusion

The proposed research investigates and successfully executes the analytical solution by reduced differential transforms method (RTDM) on the fractional Zakharov-Kuznetsov equation. The method was tested on two illustrative problems and validated through numerical and graphical results and establish more consistency as compared to another numerical method, RDTM is applied to express without using discretization, linearization, transformation, and limiting conditions. Hence, conclude that RDTM is rapidly converged, convenient to apply, and highly accurate for the various problems of engineering and science.

6.1.Limitations

Despite these advantages, the method has certain restrictions. FRDTM relies on the analyticity of the function, which may limit its applicability to equations with discontinuities or singularities. The recursive process can also become cumbersome for highly nonlinear systems or higher-dimensional models. Furthermore, convergence rates may depend on the choice of initial conditions and parameter values, requiring careful consideration in practical applications.

6.2.Future Scope

Future work can address these limitations in several ways. Extending FRDTM to handle more complex boundary conditions and singularities would broaden its applicability. Hybrid approaches that combine FRDTM with numerical schemes such as spectral or finite element methods may enhance efficiency for large-scale problems. Another promising direction is the application of FRDTM to stochastic, quantum, or multi-fractional extensions of the Zakharov–Kuznetsov equation, which are increasingly relevant in plasma physics and nonlinear dynamics. Finally, comparative studies with recently developed techniques, such as Laplace–Residual Power Series methods or Elzaki transforms, could provide deeper insights into the strengths and weaknesses of FRDTM.

In conclusion, FRDTM provides a simple, accurate, and efficient approach for solving fractional nonlinear equations, and with further development, it has the potential to become a widely used tool in applied mathematics, physics, and engineering.

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